Holyoak, K. J. (2012). Analogy and relational reasoning. In K. J. Holyoak & R. G. Morrison (Eds.), The Oxford handbook of thinking and reasoning (pp. 234-259). New York: Oxford University Press.

CHAPTER 13

Analogy and Relational Reasoning

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Abstract

Analogy is an inductive mechanism based on structured comparisons of mental representations. It is an important special case of role-based relational reasoning, in which inferences are generated on the basis of patterns of relational roles. Analogical reasoning is a complex process involving retrieval of structured knowledge from long-term memory, representing and manipulating role-filler bindings in working memory, identifying elements that play corresponding roles, generating new inferences, and learning abstract schemas. For empirical analogies, analogical inference is guided by causal knowledge about how the source analog operates. Simpler types of relation-based transfer can be produced by relational priming. Human analogical reasoning is heavily dependent on working memory and other executive functions supported by the prefrontal cortex, with the frontopolar subregion being selectively activated when multiple relations must be integrated to solve a problem.

Key Words: analogy, role-based relational reasoning, IQ, metaphor, induction, neuroimaging, frontal cortex, symbolic connectionism, mapping, retrieval, inference, schemas, System 1, System 2, causal models, relational priming, cognitive development

Introduction

Two situations are analogous if they share a common pattern of *relationships* among their constituent elements, even though the elements themselves differ across the two situations. Identifying such a common pattern requires *comparison* of the situations. Analogy involves some of the same processes as do judgments of similarity (see Goldstone & Son, Chapter 10). Typically one analog, termed the *source* or *base*, is more familiar or better understood than the second analog, termed the *target*. By "better understood," we mean that the reasoner has prior knowledge about functional relations *within* the source analog—beliefs that certain aspects of the source have causal, explanatory, or logical connections to other aspects (Hesse, 1966). This asymmetry in initial knowledge provides the basis for analogical transfer—using the source to generate inferences about the target. For example, the earliest major scientific analogy, dating from the era of imperial Rome (see Holyoak & Thagard, 1995), led to a deeper understanding of sound (the target) in terms of water waves (the source). Sound is analogous to water waves in that sound exhibits a pattern of behavior corresponding to that of water waves: propagating across space with diminishing intensity, passing around small barriers, rebounding off of large barriers, and so on. The perceptual features are very different (water is wet, air is not), but the underlying pattern of relations among the elements is similar. In this example, like most analogies involving empirical phenomena, the key functional relations involve causes and their effects (see Cheng & Buehner, Chapter 12). By transferring knowledge about causal relations, the analogy provides a new explanation of why various phenomena occur (see

Lombrozo, Chapter 14). Analogy is an inductive process, and hence analogical inferences are inevitably uncertain. The wave analogy for sound proved successful; an alternative "particle" analogy did not.

In this chapter I will focus on analogy as a key example of the broader concept of *role-based relational reasoning*. After a brief review of the history of research on analogy and related concepts, such as metaphor, I will describe current views using the framework of Marr's (1982) levels of analysis. Next, I will survey research on major subprocesses (retrieval, mapping, inference, and schema induction). This review includes both intentional and unintentional types of relational transfer, and the development of analogical abilities over the course of childhood. Finally, again applying Marr's framework, I will consider open issues and directions for future research.

Role-Based Relational Reasoning

Analogy is a prime example of role-based relational reasoning (Penn, Holyoak, & Povinelli, 2008), as its full power depends on explicit relational representations (see Doumas & Hummel, Chapter 5). Such representations distinguish relational roles from the entities that fill those roles, while coding the bindings of entities to their specific roles. Humans are capable of making inferences about entities that cannot be reliably assigned to relational roles solely on the basis of perceptual properties. In the context of the original wave analogy, water is similar to air because each serves as a medium for the transmission of waves. The wave analogy was later extended from transmission of sound to transmission of light, and ultimately it developed into an abstract *schema*, or relational category. As another example of a relational category, something fills the role of "barrier" if it blocks the passage of something else, regardless of what type of entity the "barrier" is (perhaps a landslide, perhaps poverty). If something is known to be a barrier, its binding to that relational role is enough to infer that its removal would end the blockage. Whether any other species is capable of role-based relational reasoning is a matter of debate (see Penn & Povinelli, Chapter 27).

As the earlier examples illustrate, role-based relational reasoning is broader than reasoning by analogy between specific cases (Halford, Wilson, & Phillips, 2010). More general concepts and categories are often defined at least in part by relations (e.g., *barrier, parent*, *catalyst*; see Markman & Stilwell, 2001; also Rips et al., Chapter 11). Reasoning based on rules (Smith, Langston, & Nisbett, 1992), including deductive inference (see Evans, Chapter 8; Johnson-Laird, Chapter 9), also depends critically on relations. The core property of role-based relational reasoning is that inferences about elements depend on commonalities (and sometimes differences) in the roles they play, rather than solely on perceptual features of individual elements. Although various types of inferences have this basic character, analogical inferences are especially flexible, as I will discuss in more detail later.

Functions and Processes of Analogical Reasoning

The content of analogical reasoning is extremely diverse (Holyoak & Thagard, 1995). Analogies have figured prominently in science (see Dunbar & Klahr, Chapter 35) and mathematics (Pask, 2003), and they are often used in everyday problem solving (see Bassok & Novick, Chapter 21) as well as creative cognition (Smith & Ward, Chapter 23). In legal reasoning, the use of legal precedents (relevant past cases) to help decide a new case is a special case of analogical reasoning (see Spellman & Schauer, Chapter 36). Analogies can function to sway emotions (Goode, Dahl, & Moreau, 2010; Thagard & Shelley, 2001), to influence political views (Blanchette & Dunbar, 2001; Khong, 1992), to guide consumer decisions (Markman & Loewenstein, 2010; see Lowenstein, Chapter 38), and to teach mathematics (Richland, Zur, & Holyoak, 2007). Analogy is sometimes used as part of a rational argument (Bartha, 2010; see Hahn & Oaksford, Chapter 15), using systematic connections between the source and target to generate and support plausible (though fallible) inferences about the latter.

Figure 13.1 sketches the major component processes in analogical transfer (see Carbonell, 1983; Gentner, 1983; Gick & Holyoak, 1980, 1983; Novick & Holyoak, 1991). Typically, a target situation serves as a retrieval cue for a potentially useful source analog. It is then possible to establish a *mapping*—a set of systematic correspondences that serve to align the elements of the source and target. Based on the mapping, coupled with the relevance relations within the source, it is possible to elaborate the representation of the target and derive new inferences. In the aftermath of analogical reasoning about a pair of cases, some form of relational generalization may take place yielding a more abstract schema for a category of situations (as in the case of the evolving "wave" concept), of which the source and target are both instances.

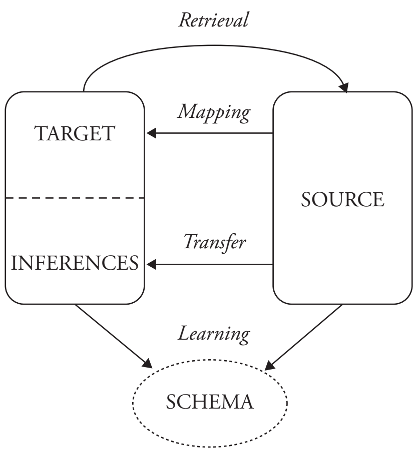

Fig. 13.1 Major components of analogical reasoning.

A Capsule History

The history of the study of analogy includes three interwoven streams of research, which respectively emphasize analogy in relation to psychometric measurement of intelligence, to metaphor and language, and to the representation of knowledge.

Psychometric Tradition

Work in the psychometric tradition focuses on four-term or "proportional" analogies, in the form A:B::C:D, such as HAND:FINGER::FOOT:?, where the problem is to infer the missing D term (TOE) that is related to C in the same way B is related to A. The pair A:B thus plays the role of source analog, and C:D that of target. Proportional analogies were discussed by Aristotle (see Hesse, 1966), and in the early decades of modern psychology became a centerpiece of efforts to define and measure intelligence. Charles Spearman (1923, 1927) argued that the best account of observed individual differences in cognitive performance was based on a general or *g* factor, with the remaining variance being unique to the particular task. He reviewed several studies that revealed high correlations between performance in solving analogy problems and the *g* factor. Spearman's student John C. Raven (1938) developed the Raven's Progressive Matrices Test (RPM), which requires selection of a geometric figure to fill an empty cell in a two-dimensional matrix (typically 3 x 3) of such figures. Much like a geometric proportional analogy, the RPM requires participants to extract and apply information based on visuospatial relations. (See Hunt, 1974, and Carpenter, Just, & Shell, 1990, for analyses of strategies for solving RPM problems.) The RPM proved to be an especially pure measure of *g*.

Raymond Cattell (1971), another student of Spearman, elaborated his mentor's theory by distinguishing between two components of *g*: *crystallized* intelligence, which depends on previously learned information or skills, and *fluid* intelligence, which involves reasoning with novel information. As a form of inductive reasoning, analogy would be expected to require fluid intelligence. Cattell confirmed Spearman's (1946) observation that analogy tests and the RPM provide sensitive measures of *g*, clarifying that they primarily measure fluid intelligence (although verbal analogies based on difficult vocabulary items also depend on crystallized intelligence). Figure 13.2 graphically depicts the centrality of RPM performance in a space defined by individual differences in performance on various cognitive tasks. Note that numerical, verbal, and geometric analogies cluster around the RPM at the center of the figure.

Because four-term analogies and the RPM are based on small numbers of relatively well-specified elements and relations, it is possible to systematically manipulate the complexity of such problems and analyze performance (based on response latencies and error rates) in terms of component processes (e.g., Mulholland, Pellegrino, & Glaser, 1980; Sternberg, 1977). The earliest computational models of analogy were developed for four-term analogy problems (Evans, 1968; Reitman, 1965). The basic components of these models were elaborations of those proposed by Spearman (1923), including encoding of the terms, accessing a relation between the A and B terms, and evoking a comparable relation between the C and D terms. As we will discuss later, four-term analogies and the RPM have proved extremely useful in recent work on the cognitive neuroscience of analogy.

Metaphor

Analogy is closely related to metaphor and related forms of symbolic expression that arise in everyday language (e.g., "the evening of life," "the idea blossomed"), in literature (Holyoak, 1982), the arts, and cultural practices such as ceremonies (see Holyoak & Thagard, 1995, ch. 9). Like analogy in general, metaphors are characterized by an asymmetry between target (conventionally termed "tenor") and source ("vehicle") domains (e.g., the target/tenor in "the evening of life" is life, which is understood in terms of the source/vehicle of time of day). In addition, a mapping (the "grounds" for the metaphor) connects the source and target,

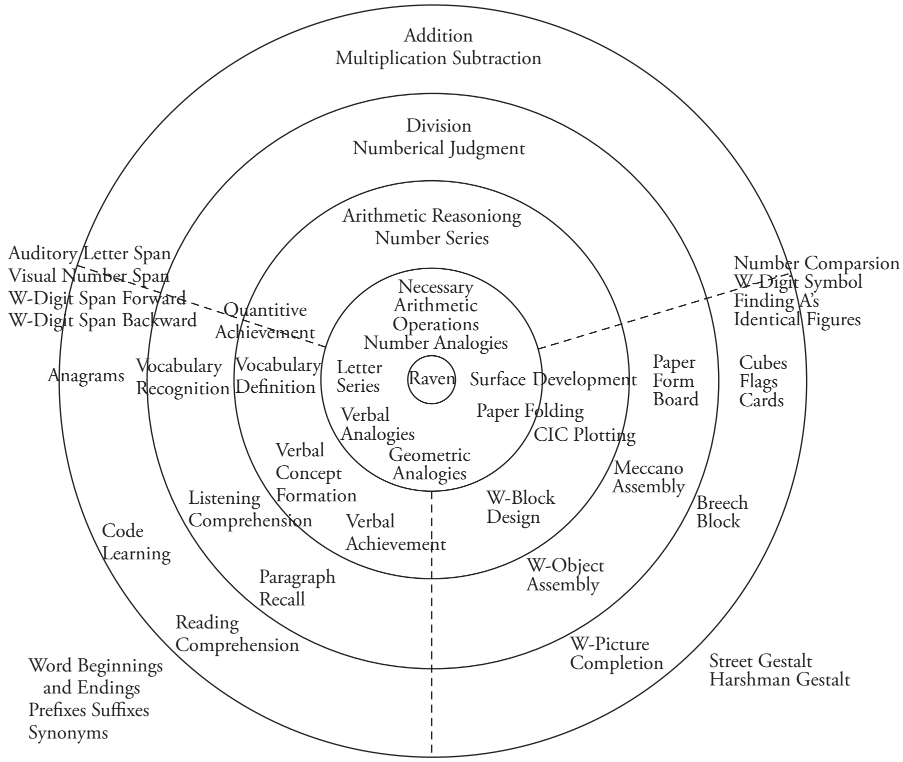

Fig. 13.2 Multidimensional scaling solution based on intercorrelations among the Raven's Progressive Matrices test, analogy tests, and other common tests of cognitive function. (Reprinted by permission from Snow, Kyllonen, & Marshalek, 1984, p. 92.)

allowing the domains to interact to generate a new conceptualization (Black, 1962). Metaphors are a special kind of analogy, in that the source and target domains are always semantically distant (Gentner, 1982; Gentner, Falkenhainer, & Skorstad, 1988), and the two domains are often blended rather than simply mapped (e.g., in "the idea blossomed," the target is directly described in terms of an action term derived from the source). In addition, metaphors are often combined with other symbolic "figures," especially metonymy (substitution of an associated concept). For example, "sword" is a metonymic expression for weaponry, derived from its ancient association as the prototypical weapon; "Raising interest rates is the Federal Reserve Board's sword in the battle against inflation" extends the metonymy into metaphor.

Fauconnier and Turner (1998; Fauconnier, 2001) have analyzed complex conceptual blends that are akin to metaphor. A typical example is a description of the voyage of a modern catamaran sailing from San Francisco to Boston, which was attempting to beat the speed record set by a clipper ship that had sailed the same route over a century earlier. A magazine account written during the catamaran's voyage said the modern boat was "barely maintaining a 4.5-day lead over the ghost of the clipper *Northern Light*." Fauconnier and Turner observed that the magazine writer was describing a "boat race" that never took place in any direct sense; rather, the writer was blending the separate voyages of the two ships into an imaginary race. The fact that such conceptual blends are so natural and easy to understand attests to the fact that people can readily comprehend novel metaphors.

Lakoff and Johnson (1980; also Lakoff & Turner, 1989) have argued that much of human experience, especially its abstract aspects, is grasped in terms of broad conceptual metaphors (e.g., events occurring in time are understood by analogy to objects moving in space). Time, for example, is understood in terms of objects in motion through space, as in expressions such as "My birthday is fast approaching" and "The time for action has arrived." (See Boroditsky, 2000, for evidence of how temporal metaphors influence cognitive judgments.) As Lakoff and Turner (1989) pointed out, the course of a life is understood in terms of time in the solar year (youth is springtime, old age is winter). Life is also conventionally conceptualized as a journey. Such conventional metaphors can still be used in creative ways, as illustrated by Robert Frost's famous poem, "The Road Not Taken":

> Two roads diverged in a wood, and I—
> I took the one less traveled by,
> And that has made all the difference.

According to Lakoff and Turner, comprehension of this passage depends on our implicit knowledge of the metaphor that life is a journey. This knowledge includes understanding several interrelated correspondences (e.g., a person is a traveler, purposes are destinations, actions are routes, difficulties in life are impediments to travel, counselors are guides, and progress is the distance traveled).

Psychological research has focused on demonstrations that metaphors are integral to everyday language understanding (Glucksberg, Gildea, & Bookin, 1982; Keysar, 1989). There has been a debate about whether metaphor is better conceptualized as a kind of analogy (Wolff & Gentner, 2000) or a kind of categorization (Glucksberg & Keysar, 1990; Glucksberg, McClone, & Manfredi, 1997). A likely resolution is that novel metaphors are interpreted by much the same processes as are analogies, whereas more conventional metaphors are interpreted as more general schemas (Gentner & Bowdle, 2008; see discussion of schemas later in this chapter).

Knowledge Representation

The most important influence on analogy research in the cognitive-science tradition has been concerned with the representation of knowledge within computational systems (see Markman, Chapter 4). Many seminal ideas were developed by the philosopher Mary Hesse (1966), who was in turn influenced by Aristotle's discussions of analogy in scientific classification and Black's (1962) interactionist view of metaphor. Hesse placed great stress on the *purpose* of analogy as a tool for scientific discovery and conceptual change, and on the close connections between causal relations and analogical mapping. In the 1970s, work in artificial intelligence and psychology focused on the representation of complex knowledge of the sort used in scientific reasoning, problem solving, story comprehension, and other tasks that require structured knowledge. A key aspect of structured knowledge is that elements can be flexibly *bound* into the roles of relations. For example, "dog bit man" and "man bit dog" have the same elements and the same relation, but the role bindings have been reversed, radically altering the overall meaning. How the mind and brain accomplish role binding is thus a central problem to be solved by any psychological theory that involves structured knowledge, including any theory of analogy (see Doumas & Hummel, Chapter 5).

In the 1980s, a number of cognitive scientists recognized the centrality of analogy as a tool for discovery, as well as its close connection with theories of knowledge representation. Winston (1980), guided by Minsky's (1975) treatment of knowledge representation, built a computer model of analogy that highlighted the importance of causal relations in guiding analogical inference. Other researchers in artificial intelligence also began to consider the use of complex analogies in reasoning and learning (Kolodner, 1983; Schank, 1982), leading to an approach to artificial intelligence termed *case-based reasoning* (Kolodner, 1993).

Meanwhile, cognitive psychologists began to consider analogy in relation to knowledge representation and eventually to integrate computational modeling with detailed experimental studies of human analogical reasoning. Gentner (1982, 1983; Gentner & Gentner, 1983) investigated the role of analogy in understanding scientific topics. She emphasized that in analogy, the key similarities involve relations that hold within the domains (e.g., the flow of electrons in an electrical circuit is analogically similar to the flow of people in a crowded subway tunnel), rather than in features of individual objects (e.g., electrons do not resemble people). Moreover, analogical similarities often depend on *higher order* relations—relations *between* relations. For example, adding a resistor to a circuit *causes* a decrease in flow of electricity, just as adding a narrow gate in the subway tunnel would decrease the rate at which people pass through (where *causes* is a higher order relation). In her structure-mapping theory, Gentner proposed that analogy entails finding a structural alignment, or mapping, between elements of the two domains. In this theory, a "good" alignment between two representational structures is characterized by a high degree of structural parallelism (consistent, one-to-one correspondences between mapped elements) and of systematicity—an implicit preference for deep, interconnected systems of relations governed by higher order relations, such as causal, mathematical, or other functional relations.

Holyoak and his colleagues (1985; Gick & Holyoak, 1980, 1983; Holyoak & Koh, 1987) focused on the role of analogy in problem solving, with a strong concern for the role of pragmatics in analogy—how causal relations that impact current goals and context guide the interpretation of an analogy. Holyoak and Thagard (1989, 1995) developed an approach to analogy in which several factors were viewed as jointly constraining analogical reasoning. According to their *multiconstraint* theory, people implicitly favor mappings that maximize structural parallelism (in agreement with Gentner's, 1983, structure-mapping theory), but that also maximize direct similarity of corresponding elements and relations, and that give priority to pragmatically important elements (i.e., those functionally related to achieving a goal). The theory further specified how the joint influence of these constraints, which often converge but sometimes conflict, might be adjudicated by a process of constraint satisfaction.

Other early work dealt with role-based relational reasoning more broadly. Gick and Holyoak (1983) provided evidence that analogical comparisons can provide the seed for forming new relational categories, by abstracting the relational correspondences between examples to form a schema for a class of problems. Halford (1993; Halford & Wilson, 1980) argued that the development of the ability to map relational structures is central to cognitive development. More generally, role-based relational reasoning came to be viewed as a central part of human induction (Holland, Holyoak, Nisbett, & Thagard, 1986; see Markman, Chapter 4; Doumas & Hummel, Chapter 5), with close ties to other basic thinking processes, including causal inference (Cheng & Buehner, Chapter 12), categorization (Rips et al., Chapter 11), and problem solving (Bassok & Novick, Chapter 21).

Relational Reasoning: Levels of Analysis

To provide an overview of current conceptions of analogy, I will focus on three questions: What are the functions of human relational reasoning, by what algorithms is it achieved, and how is it implemented in the brain? These questions instantiate Marr's (1982) three levels of analysis: computation, representation and algorithm, and implementation. Cognitive scientists and cognitive neuroscientists have addressed all three levels, focusing on analogical reasoning as a central example.

Computational Goal

At the computational level, analogies are used to achieve the goals of the reasoner (Holyoak, 1985). These goals are diverse—forming and evaluating hypotheses, solving problems, understanding new concepts, winning arguments, and so on. The focus here will be on the role of analogies and relational reasoning in pursuing what Molden and Higgins (Chapter 20) term a basic "nondirectional outcome goal": truth, coupled with relevance to current goals. The scientist seeking a good theory, the architect creating a building design that meets the client's needs, the child trying to understand how the world works, are all basically motivated to use their prior knowledge—including specific analogs—to make true and useful inferences. Of course, inductive uncertainty is inevitable, and analogies can potentially mislead. Nonetheless, the rational reasoner (see Stanovich, Chapter 22) will use analogies to reach rationally justified inferences relevant to achieving current goals—inferences that though fallible are at least plausible, and are accompanied by an appropriate sense of their degree of uncertainty (Bartha, 2010; Lee & Holyoak, 2008).

The overarching goal of making true and useful inferences underlies the major constraints on analogical inference that have been discussed in the literature. For an analogy to be successful, the structure and content of the source must provide a good model to use in elaborating the representation of the target. To the extent that the reasoner understands the functional structure of the source (i.e., what aspects depend on which other aspects), it will be possible to focus on goal-relevant information in it while "backgrounding" other details. The functional structure may take different forms. For example, in a mathematical analogy, the functional structure will involve the mathematical or logical properties that justify a conclusion (see Bartha, 2010). For empirical knowledge (i.e., knowledge about how things happen in the real world), the aim is to transfer a *causal model* (see Cheng & Buehner, Chapter 12) from source to target. In such cases the backbone of the functional structure will be cause-effect relations—"what makes what happen" in the source domain drives potential inferences about "what makes what happen" in the target.

From the perspective of Holyoak and Thagard's (1989) multiconstraint theory, the centrality of functional structure is a basic pragmatic constraint. Causal relations constitute the prime example of "higher order" relations involved in Gentner's systematicity constraint, which has been supported by experimental evidence that analogical transfer is more robust when the source includes causal structure than when it does not (Gentner & Toupin, 1986). In general, a highly systematic source will be rich in functional structure. A high degree of structural parallelism (that is, a consistent mapping between relevant elements of the source and target) is a logical requirement if the structure of the source is to provide an appropriate model for the structure of the target. Ambiguity will be minimized if the mapping is both consistent and one to one.

Holyoak and Thagard's (1989) constraint of semantic similarity—a preference for mappings in which similar objects are placed into correspondence—also follows from the overarching goal of seeking true and goal-relevant inferences. Direct semantic similarity of elements has often been termed "surface" similarity, in contrast to the "structural" variety. In fact, this contrast has been defined in (at least) two distinct ways in the analogy literature, indicating resemblances based either *(1)* on features versus relations (Gentner, 1983) or *(2)* on functionally irrelevant versus relevant elements of the analogs (Holyoak, 1985). In general, functional structure (the latter sense) will involve not only relations (i.e., predicates that take at least two arguments; see Doumas & Hummel, Chapter 5) but also those additional elements that participate in functional relations. For example, because an orange is round, fairly small, and firm (properties usually considered to be perceptual features, not relations), it could be considered analogous to a ball for purposes of playing catch. In this example, as in most simple empirical analogies, various perceptual properties participate in relevant causal relations and hence count as "structural" by the functional definition. In general, objects that share direct similarities are likely to have similar causal properties (see Rips et al., Chapter 11). Thus, while "distant" analogies between remote domains of knowledge may be especially creative (see Smith & Ward, Chapter 23), "close" analogies in which similar entities fill corresponding roles typically provide stronger support for plausible inferences (Medin & Ross, 1989; see also Koedinger & Roll, Chapter 40).

Representation and Algorithm

Role-based relational reasoning depends on the capacity to represent structured relations in terms of their roles, to represent the bindings of entities to roles, to find systematic correspondences between a source and target based on relational structure, and to use this structure to create new propositions about the target. Because of its dependence on explicit relations, all major computational models of analogical reasoning (e.g., Falkenhainer, Forbus, & Gentner, 1989; Halford, Wilson, & Phillips, 1998; Hofstadter & Mitchell, 1994; Holyoak & Thagard, 1989; Hummel & Holyoak, 1997, 2003; Keane & Brayshaw, 1988; Kokinov & Petrov, 2001) are based on some form of propositional representation capable of expressing role-filler bindings (see Markman, Chapter 4; Doumas & Hummel, Chapter 5).

Most algorithmic models of analogy, formalized as computer simulations, are based on traditional symbolic representations. Traditional connectionist systems, which lack the capacity to code variable bindings, have not been successful in modeling human-like relational reasoning (see Doumas & Hummel, Chapter 5), although they may well be applicable to simpler types of relational processing (see later discussion of relational priming), some of which appear to be within the capabilities of nonhuman animals (see Penn & Povinelli, Chapter 27). The most promising algorithmic approach to modeling human analogical reasoning in a way that makes contact with data on its neural basis is *symbolic connectionism* (Halford et al., 1998, 2010; Hummel & Holyoak, 1997, 2003; see Doumas & Hummel, Chapter 5). Models of this type aim to represent structured relations (hence "symbolic") within relatively complex neural networks (hence "connectionist"), and furthermore aim to operate within a human-like limited-capacity working memory.

The central role of working memory and related executive processes in analogical reasoning has long been supported by research in the psychometric tradition, as described earlier (see Fig. 13.2). More recent experimental work, both with normal and brain-damaged populations, has provided further evidence. For example, Waltz, Lau, Grewal, and Holyoak (2000) asked college students to map objects in a pair of pictorial scenes while simultaneously performing a secondary task designed to tax working memory (e.g., generating random digits). Adding a dual task diminished relational responses and increased similarity-based responses. A manipulation that increases people's anxiety level (performing mathematical calculations under speed pressure prior to the mapping task) yielded a similar shift in mapping responses (Tohill & Holyoak, 2000; also Feldman & Kokinov, 2009). Most dramatically, degeneration of the frontal lobes radically impairs relation-based mapping (Morrison et al., 2004; Waltz et al., 1999). These and many other findings (e.g., Cho, Holyoak, & Cannon, 2007) demonstrate that mapping on the basis of relations requires adequate working memory and attentional resources to represent and manipulate role bindings. The neural substrate of working memory is relatively well understood (see Morrison & Knowlton, Chapter 6), and its genetic mechanisms are being actively investigated (see Green & Dunbar, Chapter 7). By connecting to working memory, symbolic-connectionist models provide a potential algorithmic "bridge" between the computational and implementational levels of analysis for role-based relational reasoning.

One example of a symbolic-connectionist model of analogy is LISA (Learning and Inference with Schemas and Analogies; Hummel & Holyoak, 1997, 2003; Doumas, Hummel, & Sandhofer, 2008). LISA (described more fully by Doumas & Hummel, Chapter 5) is based on the principles of Holyoak and Thagard's (1989) multiconstraint theory of analogy. The model aims to provide a unified account of all the major components of analogical reasoning. LISA represents propositions using a hierarchy of distributed and localist units (see Fig. 13.5b; also Fig. 5.5 in Doumas & Hummel, Chapter 5). LISA includes both a long-term memory for propositions and concept meanings and a limited-capacity working memory. LISA's working memory representation, which uses neural synchrony to encode role-filler bindings, provides a natural account of the capacity limits of working memory because it is only possible to have a finite number of bindings simultaneously active and mutually *out* of synchrony.

Analog retrieval is accomplished as a form of guided pattern matching. Propositions in a *driver* analog (typically the target) generate synchronized patterns of activation on the semantic units, which in turn activate propositions in *recipients*—potential source analogs residing in long-term memory. The resulting coactivity of elements of the target and a selected source, augmented with a capacity to learn which structures in the target were coactive with which in the source, serves as the basis for analogical mapping. LISA includes a set of *mapping connections* between units of the same type (e.g., object, predicate) in separate analogs. These connections grow whenever the corresponding units are active simultaneously, and thereby permit LISA to learn correspondences between structures in separate analogs. Augmented with an algorithm for self-supervised learning, the model can generate analogical inferences based on the mapping (by using the source as the driver to generate new relational structure in the target); and further augmented with an algorithm for intersection discovery, the model provides a basis for schema induction.

LISA has been used to simulate a wide range of behavioral data on analogical reasoning in normal adults (Hummel & Holyoak, 1997, 2003). To take just one example, LISA predicts that mapping complex situations must be performed sequentially, because only a small number of propositions (two to three) can be active together in the driver analog. Mappings will be established incrementally, with early mappings constraining later ones (cf. Keane, 1997). Moreover, the success of the mapping process will depend on *which* propositions are activated together in the driver. In general, coherent, interconnected propositions (e.g., facts that are causally related) will provide more information that can be used to disambiguate a complex mapping. LISA therefore predicts that mapping will be more successful when the better-understood source acts as the driver while the less-understood target serves as recipient. Kubose, Holyoak, and Hummel (2002) showed that coherence of the driver impacts mapping, as LISA predicts. For example, people are more accurate in mapping the solved "general" story to the unsolved "tumor" problem than the reverse (see later discussion of "convergence" problems; Gick & Holyoak, 1980).

In addition to explaining phenomena concerning analogical reasoning by normal adults, LISA can account for numerous findings involving similarity judgments (Taylor & Hummel, 2009), developmental patterns (Doumas et al., 2008; Morrison, Doumas, & Richland, 2011), evidence of deficits in relational reasoning in older normal adults (Viskontas et al., 2004), and evidence of much more pronounced deficits in patients with lesions to their frontal or temporal cortex (Morrison et al., 2004).

Neural Substrate of Relational Reasoning

The implementation of role-based relational reasoning in the human brain involves a broad interconnected network of brain regions (see Morrison & Knowlton, Chapter 6). Although it is generally simplistic to identify cognitive functions with specific brain regions, several regions play major roles. The prefrontal cortex (PFC) is of central importance. The basic processes of the LISA model are closely related to known functions of PFC, in particular rapid learning (e.g., Asaad et al., 1998; Cromer, Machon, & Miller, 2011) and inhibitory control (e.g., Miller & Cohen, 2001). Other important brain regions include the hippocampus (critical for storage and retrieval of episodic knowledge), the anterior temporal cortex (storage of semantic information, including semantic relations), and the parietal cortex (representation of spatial relations).

Role of Prefrontal Cortex in Relational Integration and Interference Control

The prefrontal cortex plays a central role in relational reasoning. It has been argued that this area underlies the fluid component of Spearman's *g* factor in intelligence (Duncan et al., 2000), and it supports the executive functions of working memory and cognitive control. With respect to relational reasoning, the PFC is critical in the maintenance and active manipulation of relations and role bindings (Knowlton & Holyoak, 2009; Robin & Holyoak, 1995). Waltz et al. (1999) found that patients with frontal-lobe damage showed a marked deficit in solving problems of the Raven's-matrix type problems that required integration of two relations compared to normal controls and patients with anterior temporal lobe damage. The frontal-lobe patients performed comparably to the other groups on less complex problems that could be solved using zero or one relation. These findings imply that prefrontal cortex is critical for the integration of multiple relations.

Other neuropsychological studies have examined the role of the PFC in controlling interference from distracting information during analogical reasoning. Morrison et al. (2004) tested patients with either frontal or temporal damage, as well as age-matched controls, on a verbal analogy task. Four-term analogy problems of the form A:B::C:D or D' were employed, where D is the analogical answer and D' is a nonanalogical foil. A semantic facilitation index (SFI) was calculated for each problem to characterize the association of the correct relational pair (C:D) relative to the distractor pair (C:D'). For example, for the problem *play:game::give:* (*party* or *take*), the C:D pair (*give:party*, the correct analogical answer) is less associated than is the C:D' pair (*give:take*, the nonanalogical foil), yielding a negative SFI for the problem. The problems were divided into those with negative SFI, neutral SFI, and positive SFI in order to examine the effect of semantic interference on the ability to identify the analogical answer. Frontal patients were selectively impaired in the negative SFI condition relative to the positive and neutral SFI conditions, consistent with the hypothesis that the frontal cortex is necessary for control of interference. In contrast, temporal patients showed a more uniform decline in verbal analogy performance across all three conditions, due to their loss of the conceptual information necessary to encode the relations in the analogy problem. Using four-term picture analogies, Krawczyk et al. (2008) also found that frontal patients are especially impaired on problems that include semantically related distractors.

Functional Decomposition of Prefrontal Cortex in Reasoning Tasks

Several neuroimaging studies using functional magnetic resonance imaging (fMRI) have manipulated relational complexity using variants of Raven's Progressive Matrices (RPM) problems, similar to those used by Waltz et al. (1999) in their neuropsychological studies. For matrix problems, relational integration has been shown to consistently activate prefrontal regions. In particular, bilateral middle and inferior frontal gyri, as well as parietal and occipital regions, have been found to increase activity when multiple relations must be integrated in order to arrive at a solution, compared to problems that require processing of only a single relation (Christoff et al., 2001; Kroger et al., 2002).

Among these regions, which constitute a network commonly activated in visuospatial working memory tasks, the activation pattern of the most anterior part of the PFC (termed *frontopolar*, or *rostrolateral*) has been particularly noteworthy. Christoff et al. (2001) found that the left frontopolar region remained preferentially activated even after controlling for the influence of increased problem-solving time (also Kroger et al., 2002). Similarly, studies of verbal analogical reasoning have distinguished neural substrates of reasoning from semantic processing demands within working memory. Activation in the left frontopolar region increases selectively when making judgments of analogical similarity compared to processing of semantic associations or categories (Bunge et al., 2005; Green et al., 2006; Wendelken et al., 2008). Moreover, frontopolar activation selectively increases when the semantic distance between the A:B and C:D pairs in a "true/false" verbal analogy problem is increased (Green et al., 2010), a manipulation that may increase the demands on relational processing.

Thus, based on a substantial body of findings involving solution of different types of relational reasoning problems, the frontopolar region appears to play a special role in the process of integrating multiple relational representations to arrive at a solution. Other subregions of PFC subserve additional processes involved in relational reasoning. Cho et al. (2010)

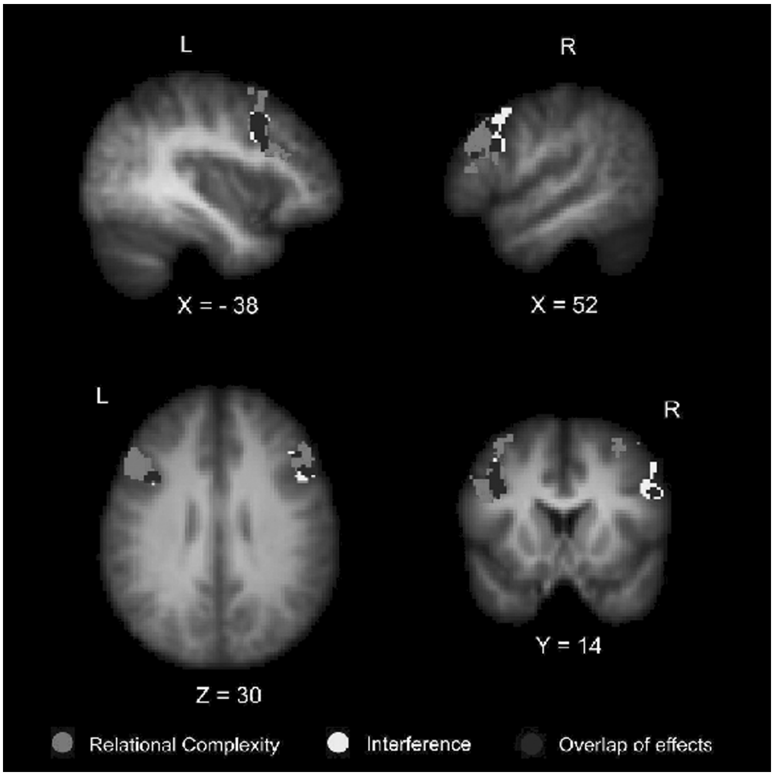

Fig. 13.3 Neuroimaging results from Cho et al. (2010). Regions showing the main effects of relational complexity (shown in red), interference (shown in yellow; small volume corrected, uncorrected cluster-forming threshold T > 2.3, corrected cluster extent significance threshold, $p < .05$), and regions where main effects overlapped (blue) within an a priori defined anatomical ROI mask of the bilateral MFG and IFG pars opercularis and pars triangularis. R, right; L, left. Coordinates are in MNI space (mm). (Reprinted by permission.) See color figure.

performed an fMRI study using four-term analogy problems based on cartoon figures and observed a partial dissociation between cortical regions sensitive to increase in demands on integration of multiple goal-relevant relations versus control of interference from goal-irrelevant relations (see Fig. 13.3). Problems requiring greater interference control selectively activated portions of the inferior frontal gyrus.

Component Processes of Analogical Reasoning

Let us now consider the subprocesses of relational reasoning in greater detail. A canonical instance of analogical reasoning involves *(1)* using retrieval cues provided by the target situation to access one or more source analogs in memory, *(2)* finding a mapping between a source and target, *(3)* using the mapping together with the functional structure of the source to make inferences about the target, and *(4)* generalizing the functional structure of the analogs (see Fig. 13.1).

A Paradigm for Studying Analogical Transfer

To study the entire process of analogical reasoning, including the retrieval of a source analog from long-term memory, a key requirement is to ensure that one or more source analogs are in fact potentially available to the reasoner. Gick and Holyoak (1980, 1983) introduced a general laboratory paradigm for investigating analogical transfer in the context of problem solving. The basic procedure was to first provide people with a source analog in the guise of some incidental context, such as an experiment on "story memory." Later, participants were asked to solve a problem that was in fact analogous to the story they had studied earlier. The questions of central interest were *(1)* whether people would spontaneously notice the relevance of the source analog and use it to solve the target problem, and *(2)* whether they could solve the analogy once they were cued to consider the source. Spontaneous transfer of the analogous solution implies successful retrieval and mapping; cued transfer implies successful mapping once the need to retrieve the source has been removed.

The source analog used by Gick and Holyoak (1980) was a story about a general who is trying to capture a fortress controlled by a dictator and needs to get his army to the fortress at full strength. Since the entire army could not pass safely along any single road, the general sends his men in small groups down several roads simultaneously. Arriving at the same time, the groups join up and capture the fortress.

A few minutes after reading this story under instructions to read and remember it (along with two other irrelevant stories), participants were asked to solve a tumor problem (Duncker, 1945), in which a doctor has to figure out how to use rays to destroy a stomach tumor without injuring the patient in the process. The crux of the problem is that it seems that the rays will have the same effect on the healthy tissue as on the tumor—high intensity will destroy both, low intensity neither. The key issue is to figure out how the rays can be made to selectively impact the tumor while sparing the surrounding tissue. The source analog, if it can be retrieved and mapped, can be used to generate a "convergence" solution to the tumor problem, one that parallels the general's military strategy: Instead of using a single high-intensity ray, the doctor could administer several low-intensity rays at once from different directions. In that way each ray would be at low intensity along its path, and hence harmless to the healthy tissue, but the effects of the rays would sum to achieve the effect of a high-intensity ray at their focal point, the site of the tumor.

When Gick and Holyoak (1980) asked college students to solve the tumor problem, without a source analog, only about 10% of them produced the convergence solution. When the general story had been studied, but no hint to use it was given, only about 20% of participants produced the convergence solution. In contrast, when the same participants were then given a simple hint that "you may find one of the stories you read earlier to be helpful in solving the problem," about 75% succeeded in generating the analogous convergence solution. In other words, people often fail to notice superficially dissimilar source analogs that they could readily use. On occasions when a person did not notice the relevance of the remote source analog, he or she sometimes reported a feeling of insight (see van Steenburgh et al., Chapter 24).

Accessing Analogs in Long-Term Memory

This gap between the difficulty of retrieving remote analogs and the relative ease of mapping them has been replicated many times, both with adults (Gentner, Rattermann, & Forbus, 1993; Holyoak & Koh, 1987; Ross, 1987, 1989; Spencer & Weisberg, 1986) and with young children (Chen, 1996; Holyoak, Junn, & Billman, 1984; Tunteler & Resing, 2002). When analogs must be cued from long-term memory, cases from a domain similar to that of the cue are retrieved much more readily than cases from remote domains (Keane, 1987; Seifert, McKoon, Abelson, & Ratcliff, 1986). For example, Keane (1987) measured retrieval of a convergence analog to the tumor problem when the source analog was studied 1–3 days prior to presentation of the target radiation problem. Keane found that 88% of participants retrieved a source analog from the same domain (a story about a surgeon treating a brain tumor), whereas only 12% retrieved a source from a remote domain (the general story). This difference in ease of access was dissociable from the ease of postaccess mapping and transfer, as the frequency of generating the convergence solution to the radiation problem once the source analog was cued was high and equal (about 86%) regardless of whether the source analog was from the same or a different domain.

The "retrieval gap" found in experimental studies of analogy is consistent with the obvious differences in the computational requirements of retrieval versus mapping. When attention is focused on two analogs, their representations will be held in working memory and explicit comparison processes can operate on relations. By definition, the question of which situations ought to be compared has already been answered. In contrast, retrieval is wide open—anything in long-term memory might potentially be relevant. As noted earlier, direct similarity of objects in the source and target is often a valid predictor of the inferential usefulness of the source. Additionally, focusing on relations in the target as retrieval cues places greater demands on working memory (see Morrison & Knowlton, Chapter 6).

Nonetheless, there is strong evidence that relational structure does play an important role in guiding analogical retrieval, both in the context of problem solving (Holyoak & Koh, 1987; Ross, 1987, 1989) and story reminding (Wharton et al., 1994; Wharton, Holyoak, & Lange, 1996). The influence of relational correspondences is greater when some degree of direct similarity of objects is also present, and when the relational correspondences favor one potential source over a nonrelational candidate competing for retrieval from long-term memory (Wharton et al., 1994). Interestingly, retrieval of verbal analogs (in the form of proverbs) is more successful when the analogs are presented in spoken rather than written form (Markman, Taylor, & Gentner, 2007), perhaps because listening imposes a reduced processing load relative to reading. In addition, domain experts (who are more likely to focus on relevant relations as retrieval cues) are more likely than novices to access remote source analogs based on relational correspondences (Novick, 1988; Novick & Holyoak, 1991). Other evidence indicates that having people generate example cases, as opposed to simply asking them to remember cases presented earlier, can enhance structure-based access to source analogs (Blanchette & Dunbar, 2000).

Unintended Memory Activation and Relational Priming

In most of the experiments discussed so far, participants were explicitly asked to remember analogous situations stored in memory when cued with an analog, and hence were clearly aware when retrieval took place. In others (e.g., Gick & Holyoak, 1980) retrieval was not explicitly requested, but participants generally seemed to be aware of using a source analog to solve the target problem (when they in fact did so). Under some circumstances, however, people may use relations as cues to access information in long-term memory even when they have not been asked to do so. Moreover, in some cases they may not be aware that a previously encountered analog is guiding their current processing of a new example. Schunn and Dunbar (1996) performed a study in which during an initial session involving a problem in biochemistry, some subjects learned that addition of an inhibitory enzyme decreased virus reproduction. In a subsequent session the following day these same subjects were asked to solve a molecular-genetics problem, which involved an analogous inhibitory gene. Schunn and Dunbar found that subjects who had been exposed to the concept of inhibition in the initial session were more likely than control subjects to develop a solution based on inhibition for the transfer problem, even though experimental subjects evinced no signs of awareness that the earlier virus problem had influenced their solution to the gene problem. Similarly, Day and Goldstone (2011) found that participants were able to transfer strategies learned from a perceptually concrete simulation of a physical system to a task with very dissimilar content and appearance. Although recognition of the analogy between the tasks was associated with better overall performance, transfer (i.e., application of an analogous strategy) was not related to such recognition (see also Day & Gentner, 2007; Wharton & Lange, 1994).

Such apparently unintended transfer likely involves a different mechanism than does deliberate analogical mapping and inference. As we will see later, intentional relational transfer makes heavy demands on working memory and appears to be a paradigmatic example of what is sometimes termed *explicit* or System 2 processing (see Evans, Chapter 8). But as Schunn and Dunbar (1996) argued, some forms of relational transfer may be more akin to *priming*, typically considered an example of *implicit* or System 1 processing. Spellman, Holyoak, and Morrison (2001) demonstrated rapid priming based on semantic relations, using both naming and lexical decision paradigms. For example, participants were able to identify a related pair such as BEAR–CAVE as words more quickly when preceded (400 msec earlier) by BIRD–NEST (same relation) as opposed to BIRD–DESERT (unrelated). Note that although the pairs BIRD–NEST and BEAR–CAVE share the same relation ("lives in"), the objects themselves are not especially similar. Spellman et al. found that it was necessary to explicitly tell participants to pay attention to relations in order to obtain relation-based priming. However, such instructions were not essential in similar paradigms when the prime was presented for a longer duration and a relational judgment about the prime was required (Estes, 2003; Estes & Jones, 2006; see also Allen, Ibara, Seymour, Cordova, & Botvinick, 2010).

Relational priming may be especially potent when its application to objects of a certain type is overlearned. For example, Bassok, Chase, and Martin (1998) demonstrated that people are sensitive to what they termed *semantic alignment* of objects with the mathematical operation of addition. These investigators found that two sets of objects that each belong to the same general category (e.g., cats and dogs, both of which are animals) could readily be aligned with the addends of addition, whereas sets of objects that are functionally related (e.g., birds and cages) proved to be far less natural as addends. Extending this finding, Bassok, Pedigo, and Oskarsson (2008) showed that automatic activation of basic arithmetic facts (e.g., $3 + 4 = 7$) is modulated by prior presentation of aligned versus nonaligned word pairs (see also Fisher, Bassok, & Osterhout, 2010). The apparent automaticity of this phenomenon is consistent with people's extensive experience in using the addition operation to solve problems involving a variety of semantically aligned object sets (e.g., blue and red marbles, cars and trucks, cupcakes and brownies). Importantly, the priming effects observed in these studies did not in general aid in task performance, suggesting that priming was automatic.

Longer term priming may have contributed to the apparent unintended transfer effects observed in studies such as those of Schunn and Dunbar (1996) and Day and Goldstone (2011), as the source analog was itself processed deeply to solve a problem. Unintended transfer may not require developing a systematic mapping of the source to the target. Rather, more piecemeal transfer may occur based on activation of one or more key relational concepts (e.g., the concept of "inhibition" in the Schunn and Dunbar study).

Mapping

Mapping—the process of identifying corresponding elements of the source and target analogs—plays a central role in analogical inference. For meaningful situations such as problems or stories, adults with intact executive functions typically are able to establish correspondences based primarily on relational roles, even when direct similarity of mapped elements is low or uninformative (Gentner & Gentner, 1983; Gick & Holyoak, 1980, 1983).

ALIGNABILITY AND ATTENTIONAL FOCUS

Markman and Gentner (1993; for a review see Gentner & Markman, 1997) drew an important distinction between commonalities (the shared properties of mapped elements), alignable differences (differences between mapped elements), and nonalignable differences (differences between analogs involving unmapped elements). In Figure 13.4, for example, the car in the top picture can be mapped to the boat in the bottom picture based on their common roles (vehicles being towed). The car and boat also exhibit alignable differences. In contrast, the parking meter in the top picture has no clear corresponding element in the bottom picture, and hence it constitutes a nonalignable difference.

The basic impact of analogical mapping is to focus attention on the commonalities and (usually to a lesser extent) the alignable differences, while backgrounding the nonalignable differences. Gentner and Markman (1994) gave college students word pairs and asked them to list one difference each for as many pairs as possible under time pressure. The participants produced many more alignable than nonalignable differences. Contrary to

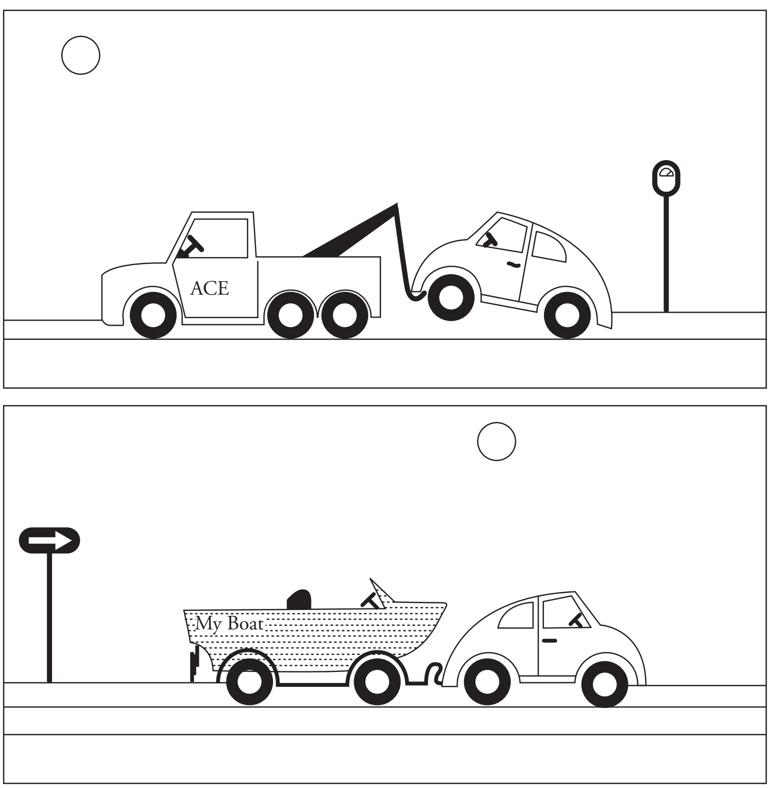

Fig. 13.4 Pictures illustrating types of analogy-based similarities and differences (Markman & Gentner, 1996). (Pictures courtesy of Art Markman.)

the commonsense idea that differences will be easier to find for dissimilar concepts, participants were actually more fluent in stating a difference for pairs of similar, alignable concepts (e.g., *hotel – motel*) than for dissimilar, nonalignable concepts (e.g., *kitten – magazine*), suggesting that the comparison process made the alignable differences especially salient. Relative to nonalignable differences, alignable differences have stronger effects on the perception of overall similarity (Markman & Gentner, 1996; see Goldstone & Son, Chapter 10), are more memorable (Markman & Gentner, 1997), and have a greater impact on choices made in a decision task (Markman & Medin, 1995).

COHERENCE IN ANALOGICAL MAPPING

The key idea of Holyoak and Thagard's (1989) multiconstraint theory of analogy is that several different kinds of constraints—similarity, structure, and purpose—all interact to determine the optimal set of correspondences between source and target. A good analogy is one that appears *coherent*, in the sense that multiple constraints converge on a solution that largely satisfies all constraints (Thagard, 2000). When constraints conflict, mappings may be ambiguous. For example, the two pictures shown in Figure 13.4 include a *cross-mapping*—the car in the top picture maps to the boat in the bottom picture on the basis of relational roles (both are vehicles being towed), but to the car in the bottom picture on the basis of direct similarity. Situations involving cross mappings are especially difficult, more so than analogies with less semantic overlap (Gentner & Toupin, 1986; Ross, 1989). Implicit cross mappings can also interfere with students' understanding of the intended interpretation of graphs and similar visuospatial representations (Gattis & Holyoak, 1996).

Comparisons based on perceptually rich stimuli, which afford an abundance of direct similarities between objects, typically lead to a lower frequency of relational responses relative to comparisons based on perceptually sparse stimuli (Markman & Gentner, 1993). In general, manipulations that increase attention to relations tend to encourage a relation-based response. For example, Markman and Gentner found that people who mapped three objects at once were more likely to map on the basis of similar relational roles than were people who mapped just one cross-mapped object, presumably because mapping multiple objects focuses greater attention on relations among them. Relational language is an especially important influence on mapping. For preschool children, Loewenstein and Gentner (2005) found that explicitly describing a scene in terms of spatial relations increased the frequency of relation-based mappings (for a general discussion of language and thought, see Gleitman & Papafragou, Chapter 28).

In the absence of cross mappings, adults are generally able to integrate multiple constraints to establish coherent mappings, even for situations that are complex and somewhat ambiguous. For example, at the beginning of the first Gulf War in 1991, Spellman and Holyoak (1992) asked a group of American undergraduates a few questions to find out how they interpreted the analogy between the then-current situation in the Persian Gulf and World War II. The undergraduates were asked to suppose that Saddam Hussein, the President of Iraq, was analogous to Hitler (a popular analogy at the time). Regardless of whether they thought the analogy was appropriate, they were then asked to write down the most natural match in the World War II situation for various people and nations involved in the Gulf War, including the United States and its current President, George H. W. Bush. For those students who gave evidence that they knew the basic facts about World War II, the majority produced mappings that fell into one of two patterns, each coherent on relational grounds. Those students who mapped the United States to itself also mapped Bush to Franklin D. Roosevelt. Other students, in contrast, mapped the United States to Great Britain and Bush to Winston Churchill, the British Prime Minister (perhaps because Bush, like Churchill, led his nation and Western allies in early opposition to aggression). The analogy between the Persian Gulf situation and World War II thus generated a "bistable" mapping: People tended to provide mappings based on either of two coherent but mutually incompatible sets of correspondences.

Mapping is guided not only by relational structure and element similarity but also by the goals of the analogist (Holyoak, 1985). Particularly when the mapping is inherently ambiguous, the constraint of pragmatic centrality—relevance to goals—is critical (Holyoak, 1985). Spellman and Holyoak (1996) investigated the impact of processing goals on the mappings generated for inherently ambiguous analogies and found that mappings were predominately determined by those relations most relevant to the reasoner's goal.

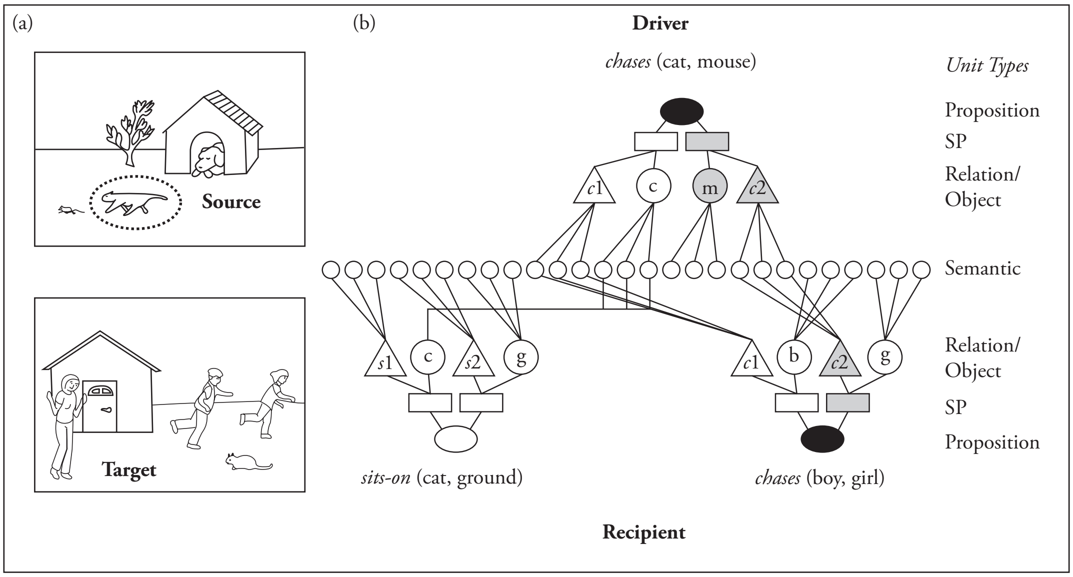

Fig. 13.5 (*A*) Example of one-relation/distractor scene-analogy problem (Richland et al., 2006); (*B*) LISA architecture as applied to this problem. In order for a reasoner to select the boy in the target as the correct analogical mapping to the cat in the source, units in the recipient representing the proposition *chases* (boy, girl) must inhibit corresponding units in the propositional structure containing the featurally similar "sitting cat" distractor. (Reprinted with permission from Morrison, Doumas, & Richland, 2011.)

DEVELOPMENTAL CHANGES IN ANALOGICAL MAPPING

Young children are particularly sensitive to direct similarity of objects. When asked to identify corresponding elements in two analogs when semantic and structural constraints conflict, their mappings are dominated by object similarity (Gentner & Toupin, 1986). The developmental transition toward greater reliance on relational structure in mapping has been termed the *relational shift* (Gentner & Rattermann, 1991). The empirical phenomenon of a relational shift is well established, but there has been some debate regarding the developmental mechanisms that may underlie it. Goswami and colleagues have argued that analogical reasoning is fundamentally available as a capacity from early infancy (and indeed, some analogical ability is apparent in 1-year-old children; Chen, Sanchez, & Campbell, 1997), but that children's analogical performance increases with age due to the accretion of knowledge about relevant relations (Goswami, 1992, 2001; Goswami & Brown, 1989). Knowledge of relations is without doubt essential for analogical reasoning. Even for adults, expertise in a domain is a predictor of superior ability to process analogies in that domain (Novick & Holyoak, 1991).

However, although accretion of knowledge is certainly an important factor, there is now evidence that maturational changes in cognitive functioning also drive developmental differences in analogical reasoning. These changes are associated with the maturation of the prefrontal cortex (see Diamond, 2002), which is not complete until adolescence. The prefrontal cortex underlies executive control (Diamond, 2006; see Morrison & Knowlton, Chapter 6), and in particular the capacity to manipulate complex information in working memory and to inhibit salient but task-inappropriate information and responses. A study by Richland, Morrison, and Holyoak (2006) illustrates how the need for working memory and inhibitory control influence the difficulty of analogical mapping at different ages. Figure 13.5a depicts an example of "scene-analogy" problems developed for use with children as young as 3–4 years old; Figure 13.5b illustrates how the LISA model (Hummel & Holyoak, 1997) would represent the mapping problem. For each pair of pictures, children were asked to identify the object in the bottom picture that "goes with" the object indicated by an arrow in the top picture. In some problems, such as that shown in Figure 13.5a, the child is confronted with a conflict between two possible answers, one relational and one based on perceptual and/or semantic similarity. The cat in the top picture perceptually resembles the cat in the bottom picture, but it plays a role (chasing a mouse) that parallels the role played by the boy in the bottom picture (chasing a girl). Richand et al. found that young children were less likely to give the relational response when an alternative based on direct similarity was available. Inhibitory control is presumably required to avoid responding on the basis of direct similarity.

In addition, relational reasoning varies in its complexity, which has been linked to the number of relational roles relevant to an inference (Halford, 1993; Halford et al., 1998). The load on working memory will be less if a single relation is sufficient to determine the role-based inference (as in the example shown in Fig. 13.5a), compared to when multiple relations must be integrated to derive the inference. Richland et al. (2006) found that preschool children gave fewer relational responses when either a similar distractor was present in the bottom picture or when two relations had to be integrated. By age 13–14 years—roughly the age at which the prefrontal cortex has undergone substantial further maturation—children reliably gave the relational response even when multiple relations had to be integrated and a similar distractor was present. Children with autism, when matched to controls on measures of executive function, show comparable trends in analogical reasoning (Dawson et al., 2007; Morsanyi & Holyoak, 2010). This pattern of analogical development is consistent with what is known about the neural basis for analogical reasoning in adults, as discussed earlier.

Analogical Inference

Analogical inference—using a source analog to form a new conjecture, whether it be a step toward solving a math problem (see Bassok & Novick, Chapter 21), a scientific hypothesis (Dunbar & Klahr, Chapter 35), a basis for deciding a legal case (Spellman & Schauer, Chapter 36), or finding a diagnosis for puzzling medical symptoms (Patel et al., Chapter 37)—is the fundamental purpose of analogical reasoning (Bartha, 2010). Mapping serves to highlight correspondences between the source and target. These correspondences provide the input to an inference engine that generates new target propositions.

The basic algorithm for analogical inference used by all major computational models has been termed "copy with substitution and generation," or CWSG (Holyoak, Novick, & Melz, 1994), and involves constructing target analogs based on unmapped source propositions by substituting the corresponding target element (if known) for each source element, and if no corresponding target element is known, postulating one as needed. CWSG allows the generation of structured propositions about the target (as opposed to simple associations) because of its reliance on variable binding and mapping. In this key respect, inference by CWSG is similar to rule-based inferences of the sort modeled by production systems (e.g., Anderson & Lebiere, 1998; see Doumas & Hummel, Chapter 5; Koedinger & Roll, Chapter 40). However, the constraints on analogical mapping are more fluid than are the typical constraints on matching in a production system. CWSG is more flexible in that unlike production rules, there is no strict division between a "left-hand side" to be matched and a "right-hand side" that creates an inference. Rather, any subset of the two analogs may provide an initial mapping, and the unmapped remainder of the source may be used to create target inferences (giving rise to the property of *omni-directional access* in analogical inference; Halford et al., 1998). Analogical inference might be described as a "strong weak method"—a domain-general method (see Bassok & Novick, Chapter 21) that can be extremely powerful if the requisite knowledge about a source analog is available (though useless if an appropriate source is lacking).

INTEGRATING ANALOGICAL INFERENCE WITH CAUSAL MODELS

The CWSG algorithm, and analogical inference in general, can fail in a variety of ways. If critical elements are difficult to map (e.g., because of strong representational asymmetries, such as those that hinder mapping a discrete set of elements to a continuous variable; Bassok & Olseth, 1995; Bassok & Holyoak, 1989), then no inferences can be constructed. If elements are mismapped, corresponding inference errors will result (Holyoak et al., 1994; Reed, 1987). Most important, the great fluidity of CWSG has its downside. Without additional constraints on when CWSG is invoked, *any* unmapped source proposition would generate an inference about the target. Such a loose criterion for inference generation would lead to rampant errors whenever the source was not isomorphic to a subset of the target; and such isomorphism will virtually never hold for problems of realistic complexity. Accordingly, additional constraints are required (Clement & Gentner, 1991; Holyoak et al., 1994; Markman, 1997).

Lassaline (1996) provided evidence of factors that appear to constrain analogical inferences. She had college students read descriptions of the properties of hypothetical animals, and then rate various possible target inferences for the probability that the conclusion would be true, given the information in the premise. Participants rated potential inferences as more probable when the source and target analogs shared more attributes, and hence were more similar. In addition, the presence of a causal relation in the source made an inference more credible. For example, if the source and target animals were both described as having a weak immune system, and for the source the weak immune system was stated to "cause" an acute sense of smell, then the inference that the target animal also has an acute sense of smell would be bolstered relative to stating only that the source animal had a weak immune system "and" an acute sense of smell. The benefit conveyed by the linking relation was reduced if it was less clearly causal ("develops before"). Lassaline's findings thus imply that although analogical inferences are influenced by the overall similarity of the analogs, causal relations in the source play an especially important role.

Work by Lee and Holyoak (2008) demonstrated the close connection between analogical inference and the operation of representations that have been termed *causal models* (Waldmann & Holyoak, 1992; see Cheng & Buehner, Chapter 12)—a network of cause-effect relations characterizing each analog. Holyoak, Lee, and Lu (2010) formalized this integration by extending a Bayesian model of causal learning (Lu et al., 2008) to deal with analogical inference. The basic idea is that for empirical analogies, the causal model of the source analog (including information about the strength distributions associated with individual causal links), coupled with the mapping of source to target, provide the input to CWSG. This procedure constrains CWSG is a way that favors accurate and useful inferences, generating as its output an elaborated causal model of the target. This causal model is then used to evaluate the probability of specific inferences about the target. By treating analogical and causal inference within a unifying theoretical framework, it proved possible to explain situations in which the strengths of inferences about the target are dissociable from the overall similarity of the source and target. Most dramatically, Lee and Holyoak showed that if the source exhibits an effect *despite* the presence of a preventive cause, then people judge the effect to be *more* likely in the target if it lacks the preventer (even though absence of the preventer reduces overall similarity of the source and target).

Rather than considering analogical inference in isolation, it is useful to view the entire transfer process as the joint product of causal learning and relational mapping: The reasoner learns the causal structure of the source, maps the source to target, applies CWSG to augment the causal model of the target, and then uses the resulting model to evaluate an open-ended range of potential inferences about the target. The model of the target generated on the basis of the source will often be imperfect, so that additional postanalogical processes of *adaptation* will be required to accommodate goal-relevant aspects of the target that are not predictable from the source (Carbonell, 1983; Holyoak et al., 1994).

Analogical Inferences as "False Memories"

An important question concerns when analogical inferences are made, and how inferences relate to facts about the target analog that are stated directly. One extreme possibility is that people only make analogical inferences when instructed to do so, and that inferences are carefully "marked" as such, so that they will never be confused with known facts about the target. At the other extreme, it is possible that some analogical inferences are triggered when the target is first processed (given that the source has been activated), and that such inferences are then integrated with prior knowledge of the target. One paradigm for addressing this issue is based on testing for false "recognition" of potential inferences in a subsequent memory test. The logic of the recognition paradigm is that if an inference has been made and integrated with the rest of the target analog, then later the reasoner will believe that the inference had been directly presented, in effect having created a "false memory" (see Brainerd & Reyna, 2005).

Early work by Schustack and Anderson (1979) provided evidence that people sometimes falsely report that analogical inferences were actually presented as facts. Blanchette and Dunbar (2002) performed a series of experiments designed to assess when analogical inferences are made. They had college students (in Canada) read a text describing a current political issue, possible legalization of marijuana use, which served as the target analog. Immediately afterward, half the students read, "The situation with marijuana can be compared to…" followed by an additional text describing the period early in the 20th century when alcohol use was prohibited. Importantly, the students in the analogy condition were not told how prohibition mapped onto the marijuana debate, nor were they asked to draw any inferences. After a delay (1 week in one experiment, 15 minutes in another), the students were given a list of sentences and were asked to decide whether each sentence had actually been presented in the text about marijuana use. The critical items were sentences such as “The government could set up agencies to control the quality and take over the distribution of marijuana.” These sentences had never been presented; however, they could be generated as analogical inferences by CWSG, based on a parallel statement contained in the source analog (“The government set up agencies to control the quality and take over the distribution of alcohol”). Blanchette and Dunbar found that students in the analogy condition said “yes” to analogical inferences about 50% of the time, whereas control subjects who had not read the source analog about prohibition said “yes” only about 25% of the time. This tendency to falsely “recognize” analogical inferences that had never been read was obtained both after long and short delays, and with both familiar and less familiar materials. Similar findings have been obtained by Perrott, Gentner, and Bodenhausen (2005).

It thus appears that when people notice the connection between a source and target, and they are sufficiently engaged in an effort to understand the target situation, analogical inferences will often be generated and then integrated with prior knowledge of the target. In some cases such transfer may be unintended and involve relational priming, as discussed earlier. At least sometimes, an analogical inference becomes accepted as a stated fact. Like relational priming, this is a case in which relational transfer does not necessarily improve performance of the target task (recognition memory). Such findings have important implications for understanding how analogical reasoning can operate as a tool for persuasion.

Relational Generalization and Schema Induction

In addition to generating local inferences about the target, analogical reasoning can give rise to relational generalizations—abstract schemas that establish an explicit representation of the commonalities between the source and target. *Comparison*—not simply passive accumulation of information about distributions of features across examples, but active generation of structural correspondences—lies at the heart of analogical reasoning. Comparison of multiple analogs can result not only in a specific mapping but also in the induction of a schema, which in turn will facilitate subsequent transfer to additional analogs. The induction of such schemas has been demonstrated in both adults (Catrambone & Holyoak, 1989; Gick & Holyoak, 1983) and young children (Brown, Kane, & Echols, 1986; Chen & Daehler, 1989; Holyoak et al., 1984; Kotovsky & Gentner, 1996; Loewenstein & Gentner, 2001; Namy & Gentner, 2002).

Comparison has been shown to guide schema formation in teaching such complex topics as negotiation strategies (Loewenstein, Thompson, & Gentner, 1999, 2003; see Loewenstein, Chapter 38). There is also evidence that comparison may play a key role in learning role-based relations (e.g., comparative adjectives such as “bigger than”) from nonrelational inputs (Doumas et al., 2008), and in language learning more generally (Gentner, 2010; Gentner & Namy, 2006). An important refinement of the use of comparison as a training technique is to provide a series of comparisons ordered “easy to hard,” where the early pairs share salient surface similarities as well as less salient relational matches, and the later pairs share only relational matches. This “progressive alignment” strategy serves to promote a kind of analogical bootstrapping, using salient similarities to aid the learner in identifying appropriate mappings between objects that also correspond with respect to their relational roles (Kotovsky & Gentner, 1996).

FACTORS THAT INFLUENCE SCHEMA INDUCTION

People are able to induce schemas by comparing just two analogs to one another (Gick & Holyoak, 1983). Indeed, people will form schemas simply as a side effect of applying one solved source problem to an unsolved target problem (Novick & Holyoak, 1991; Ross & Kennedy, 1990). In the case of problem schemas, more effective schemas are formed when the goal-relevant relations are the focus rather than incidental details (Brown et al., 1986; Brown, Kane, & Long, 1989; Gick & Holyoak, 1983). In general, any kind of processing that helps people focus on the underlying functional structure of the analogs, thereby encouraging learning of more effective problem schemas, will improve subsequent transfer to new problems. For example, Gick and Holyoak (1983) found that induction of a “convergence” schema from two disparate analogs was facilitated when each story stated the underlying solution principle abstractly: “If you need a large force to accomplish some purpose, but are prevented from applying such a force directly, many smaller forces applied simultaneously from different directions may work just as well." In some circumstances transfer can also be improved by having the reasoner generate a problem analogous to an initial example (Bernardo, 2001). Other work has shown that abstract diagrams that highlight the basic idea of using multiple converging forces can aid in schema induction and subsequent transfer (Beveridge & Parkins, 1987; Gick & Holyoak, 1983).

Although two examples can suffice to establish a useful schema, people are able to incrementally develop increasingly abstract schemas as additional examples are provided (Brown et al., 1986; Brown et al., 1989; Catrambone & Holyoak, 1989). Even with multiple examples that allow novices to start forming schemas, people may still fail to transfer the analogous solution to a problem drawn from a different domain if a substantial delay intervenes or if the context is changed (Spencer & Weisberg, 1986). Nonetheless, as novices continue to develop more powerful schemas, long-term transfer in an altered context can be dramatically improved (Barnett & Koslowski, 2002). For example, Catrambone and Holyoak (1989) gave college students a total of three convergence analogs to study, compare, and solve. The students were first asked a series of detailed questions designed to encourage them to focus on the abstract structure common to two of the analogs. After this abstraction training, the students were asked to solve another analog from a third domain (not the tumor problem), after which they were told the convergence solution to it (which most students were able to generate themselves). Finally, a week later, the students returned to participate in a different experiment. After the other experiment was completed, they were given the tumor problem to solve. Over 80% of participants came up with the converging-rays solution without any hint. As the novice becomes an expert, the emerging schema becomes increasingly accessible and is triggered by novel problems that share its structure (see Koedinger & Roll, Chapter 40). Deeper similarities have been constructed between analogous situations that fit the schema.

IMPACT OF SCHEMAS ON RELATIONAL PROCESSING

As schemas are acquired from examples, they in turn guide future analog retrieval, mapping, and inference. We have already seen how schema induction can increase subsequent transfer to novel problems (e.g., Bassok & Holyoak, 1989; Gick & Holyoak, 1983; Loewenstein et al., 2003), as well as facilitate processing of metaphors (Gentner & Bowdle, 2008). In addition, a schema induced by comparing examples can work "backward" in memory, making it easier to retrieve analogous episodes (including autobiographical memories) that had been stored before the schema was acquired (Gentner, Loewenstein, Thompson, & Forbus, 2009).

Of course, schemas are often acquired through experience outside the laboratory. Such preexisting schemas can guide the interpretation of specific examples, thereby changing analogical mapping and inference. For example, Bassok, Wu, and Olseth (1995) examined analogical reasoning with algebra word problems similar to ones studied previously by Ross (1987, 1989). Participants were shown how to compute permutations using an example (the source problem) in which some items from a set of n members were randomly assigned to items from a set of m members (e.g., how many different ways can you assign three computers to three secretaries if there are n computers and m secretaries?). Participants were then tested for their ability to transfer this solution to new target problems. The critical basis for transfer hinged on the assignment relation in each analog—that is, what kinds of items (people or inanimate objects) served in the roles of n and m. In some problems (type OP) objects (n) were assigned to people (m; e.g., "computers were assigned to secretaries"); in others (PO) people (n) were assigned to objects (m; "secretaries were assigned to computers"). Solving a target problem required the participant to map the elements of the problem appropriately to the variables n and m in the equation for calculating the number of permutations.

Although all the problems were formally isomorphic, Bassok et al. (1995) demonstrated that people will typically interpret an "assign" relation between an object and a person as one in which the person *gets* the object. Importantly, the "get" schema favors interpreting the person as the recipient of the object no matter which entity occupies which role in the stated "assign" relation. These distinct interpretations of the stated "assign" statement yielded systematic consequences for analogical mapping. Given an OP source analog, Bassok et al. found that people tended to link the "assigned" object to the "received" role (rather than the "recipient" role) of the "get" schema, which in turn was then mapped to the mathematical variable *n*, the number of "assigned" objects. As a result, when the target analog also had an OP structure, transfer was accurate (89%); but when the target was in the reversed PO structure, the object set continued to be linked to the "received" role of "get," and hence erroneously mapped to *n* (0% correct!). Bassok et al.'s findings highlight the constructive and interactive nature of relational processing (see also Hofstadter & Mitchell, 1994).

Conclusions

Analogy is an important special case of role-based relational reasoning, a psychological process that generates inferences based on patterns of relational roles. At its core, analogy depends on comparison of situations. But humans do much more than just compare two analogs based on obvious similarities between their elements. Rather, analogical reasoning is a complex process of retrieving structured knowledge from long-term memory, representing and manipulating role-filler bindings in working memory, generating new inferences, and finding structured intersections between analogs to form new abstract schemas. For empirical analogies, analogical inference is guided by causal knowledge about how the source analog operates. Simpler types of relation-based transfer can be produced by relational priming.

Symbolic-connectionist models have the greatest promise in relating relational reasoning to its neural substrate. Human analogical reasoning is heavily dependent on working memory and other executive functions supported by the prefrontal cortex, with the frontopolar subregion being selectively activated when multiple relations must be integrated to solve a problem.

Future Directions

Computational Level

Theoretical work to date has specified qualitative constraints on analogical mapping and inference, often implemented in computer simulations. Recent efforts to integrate analogical inference with Bayesian causal models (Holyoak et al., 2010) suggest that human analogical inference may be approximately normative when the analogs can be represented as simple causal networks based on binary variables. However, a full computational-level analysis of relational reasoning using the modern Bayesian framework for induction (see Griffiths et al., Chapter 3) has not yet been offered. Given representations of source and target analogs (including relevant prior knowledge), normative probability distributions for possible analogical mappings and inferences might in principle be derived. However, in practice this remains a challenging (perhaps even intractable) project, given that the types of relations involved in analogies are indefinitely diverse and include many different types of causal functions. One key requirement for applying the Bayesian framework to analogy will be greater theoretical integration of role-based relational representations with probabilistic inference.

Level of Representation and Algorithm

For over 30 years, a great deal of effort has been directed at the development of algorithmic models of analogical reasoning, formalized as computer simulations. In recent years, some of these models (based on the symbolic-connectionist framework) have begun to make contact with work on the neural substrate of relational reasoning. However, no model as yet comes close to providing a comprehensive account of how humans reason with relations.

One basic limitation is that human relational reasoning is far more flexible than any current simulation model (Bartha, 2010; Bassok et al., 1995; Hofstadter & Mitchell, 1994). The stage analysis typically used (for example, in this chapter) to organize analogical processing—retrieval, mapping, inference, schema induction (see Fig. 13.1)—is oversimplified. In everyday use of analogies, the entire process may be cyclic, interactive, and open ended. The effective representation of the source analog may be developed in the very process of reasoning with it. Multiple source analogs and schemas may be involved, some of them imagined rather than actual (as in the case of analogical "thought experiments;" see Bartha, 2010; Holyoak & Thagard, 1995). Typically there is no firm boundary around the information that counts as an individual analog. Causal knowledge, which is deeply embedded in the representation of individual cases, is almost inevitably linked to more general categories. Analogs, schemas, categories, and rules all interact with one another in the course of inductive inference (Holland et al., 1986). In addition, much more needs to be learned about how "full-blown" System 2 relational processing relates to simpler System 1 processing (e.g., relational priming).

A related limitation of current computational models of analogy is that their knowledge representations typically must be hand-coded by the modeler, whereas human knowledge representations are formed autonomously. In effect, modelers have allowed themselves an indefinite number of free parameters to facilitate data-fitting. There have been recent efforts to extend analogy models to account for how humans learn basic perceptual relations from nonrelational inputs (Doumas et al., 2008). This is a welcome development, but even here, the nonrelational inputs have themselves been hand-coded by the modelers.

Closely related to the challenge of avoiding hand-coding of representations is the need to flexibly re-represent knowledge so as to render potential analogies perspicuous. Concepts often have a close conceptual relationship with more complex relational forms (Jackendoff, 1983). For example, causative verbs such as *lift* (e.g., "John lifted the hammer") have very similar meanings to structures based on an explicit higher order relation, *cause* (e.g., "John caused the hammer to rise"). In such cases the causative verb serves as a "chunked" representation of a more elaborate predicate-argument structure. People are able to "see" analogies even when the analogs have very different linguistic forms (e.g., "John lifted the hammer in order to strike the nail" might be mapped onto "The Federal Reserve used an increase in interest rates as a tool in its efforts to drive down inflation"). A deeper understanding of human knowledge representation is a prerequisite for a complete theory of analogical reasoning.

Yet another limitation is that most research and modeling in the field of analogy has emphasized quasi-linguistic knowledge representations, but there is good reason to believe that reasoning in general has close connections to perception and action (see Hegarty & Stull, Chapter 31; Goldin-Meadow & Cook, Chapter 32). The ease of solving apparently isomorphic problems (e.g., isomorphs of the well-known Tower of Hanoi) can vary enormously depending on perceptual cues (Kotovsky & Simon, 1990). Models of analogy have not offered an adequate account of why the difficulty of solving problems and transferring solution methods to isomorphic problems is dependent on the difficulty of perceptually encoding key relations.

In addition, models of analogy have not been well integrated with models of problem solving (see Bassok & Novick, Chapter 21), despite the fact that analogy clearly affords an important mechanism for solving problems. In its general form, problem solving requires sequencing multiple operators, establishing subgoals, and using combinations of rules to solve related but nonisomorphic problems. These basic requirements are beyond the capabilities of current computational models of analogy. The integration of analogy models with models of general problem solving remains an important research goal.

Neural Implementation

The recent advances in understanding the neural substrate of relational reasoning (in particular, the roles played by specific areas of prefrontal cortex operating within broader neural circuits) have set the stage for further work on how analogies are computed by the brain (see Morrison & Knowlton, Chapter 6). For example, tasks involving analogical processing, like those designed to elicit insight (see van Steenburgh et al., Chapter 24), should prove useful in investigating connections between the neural bases of cognition and emotion. Careful studies will be required to determine how the neural systems involved in analogical reasoning relate to those involved in other forms of role-based relational reasoning, such as deductive and linguistic inferences (e.g., Monti, Parsons, & Osherson, 2009).

Given that a significant evolutionary gap may separate human role-based relational reasoning from the capabilities of other extant primate species (Penn et al., 2008), animal models may not provide fully adequate models of human reasoning. Testing the mechanisms postulated by symbolic-connectionist models (Hummel & Holyoak, 1997), such as dynamic binding controlled by synchronous neural activity in the gamma band, and rapid cortical learning of mapping connections, will require noninvasive neuroimaging techniques that provide extremely fine temporal resolution coupled with good spatial resolution (but see Lu et al., 2006, for an example of how a behavioral priming technique may be useful for assessing the role of synchrony in perceptual representation). The emerging field of cognitive neurogenetics (see Green & Dunbar, Chapter 7) will doubtless provide deeper insights into the neural basis of human analogical reasoning.

Translational Research

In parallel with continued basic research on role-based relational reasoning, we can anticipate advances in many application areas, such as the use of analogies in teaching and learning (e.g., Richland, Stigler, & Holyoak, 2012), as aids to creative design, and in applications to computer-based search algorithms. The limits of analogical applications are roughly coextensive with those of human imagination.

Acknowledgments

Preparation of this chapter was supported by grant N000140810186 from the Office of Naval Research, and grant R305C080015 from the Institute of Education Sciences. I thank Miriam Bassok, Alex Doumas, John Hummel, Art Markman, Bob Morrison, and Derek Penn for helpful discussions and suggestions.

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